Chain rule with product rule

Let's look at an example of how we might see the chain rule in conjunction with product rule.

Example

Use chain rule to find the derivative.

$$y = 8\left(6xe^x\right)^{-4}$$

Using substitution, we set $u = 6xe^x$ and use product rule to find that

$$u' = (6)\left(e^x\right) + (6x)\left(e^x\right)$$

$$u' = 6e^x + 6xe^x$$

Our original equation would then look like

$$y = 8(u)^{-4}$$

and according to power rule, the derivative would be

$$y' = -32(u)^{-5}u'$$

Back-substituting, we get

$$y' = -32\left(6xe^x\right)^{-5}\left(6e^x + 6xe^x\right)$$

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$$y' = -\frac{32\left(6e^x + 6xe^x\right)}{\left(6xe^x\right)^5}$$

$$y' = -\frac{192e^x(x+1)}{7{,}776x^5e^{5x}}$$

$$y' = -\frac{2(x+1)}{81e^{4x}x^5}$$

Let's look at another example of chain rule being used in conjunction with product rule.

Example

Use chain rule to find the derivative.

$$y = (x^2+1)^7\left(9x^4\right)$$

In this case, $u = x^2 + 1$ and $u' = 2x$.

Then the original equation is

$$y = (u)^7\left(9x^4\right)$$

and according to product rule, the derivative is

$$y' = 7(u)^6(u')\left(9x^4\right) + (u)^7\left(36x^3\right)$$

Back-substituting for u and u' gives

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$$y' = 7(x^2+1)^6(2x)\left(9x^4\right) + (x^2+1)^7\left(36x^3\right)$$

$$y' = 126x^5(x^2+1)^6 + 36x^3\left(x^2+1\right)^7$$

$$y' = 6x^3\left(x^2+1\right)^6\left[21x^2 + 6\left(x^2+1\right)\right]$$

$$y' = 6x^3\left(x^2+1\right)^6\left(21x^2 + 6x^2 + 6\right)$$

$$y' = 6x^3\left(x^2+1\right)^6\left(27x^2 + 6\right)$$

$$y' = 18x^3\left(x^2+1\right)^6\left(9x^2 + 2\right)$$

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